



USP

Surplus-Production Models

- Purpose of slides:
 - Introduction to the production model
 - Overview of different methods of fitting
 - Go over some critique of the method

- Source:
 - Haddon 2001, Chapter 10
 - Hilborn and Walters 1992, Chapter 8

- Nomenclature:
 - surplus production models = biomass dynamic models
- Simplest analytical models available
- Pool recruitment, mortality and growth into a single production function
 - The stock is thus an undifferentiated mass
- Minimum data requirements:
 - Time series of index of relative abundance
 - Survey index
 - Catch per unit of effort from the commercial fisheries
 - Time series of catch

General form of surplus production

Are related directly to Russels mass-balance formulation:

$$B_{t+1} = B_t + R_t + G_t - M_t - C_t$$

$$B_{t+1} = B_t + P_t - C_t$$

$$B_{t+1} = B_t + f(B_t) - C_t$$

- B_{t+1} : Biomass in the beginning of year $t+1$ (or end of t)
- B_t : Biomass in the beginning of year t
- P_t : Surplus production
 - the difference between production (recruitment + growth) and natural mortality
- $f(B_t)$: Surplus production as a function of biomass in the start of the year t
- C_t : Biomass (yield) caught during year t

Functional forms of surplus productions

- Classic Schaefer (logistic) form:

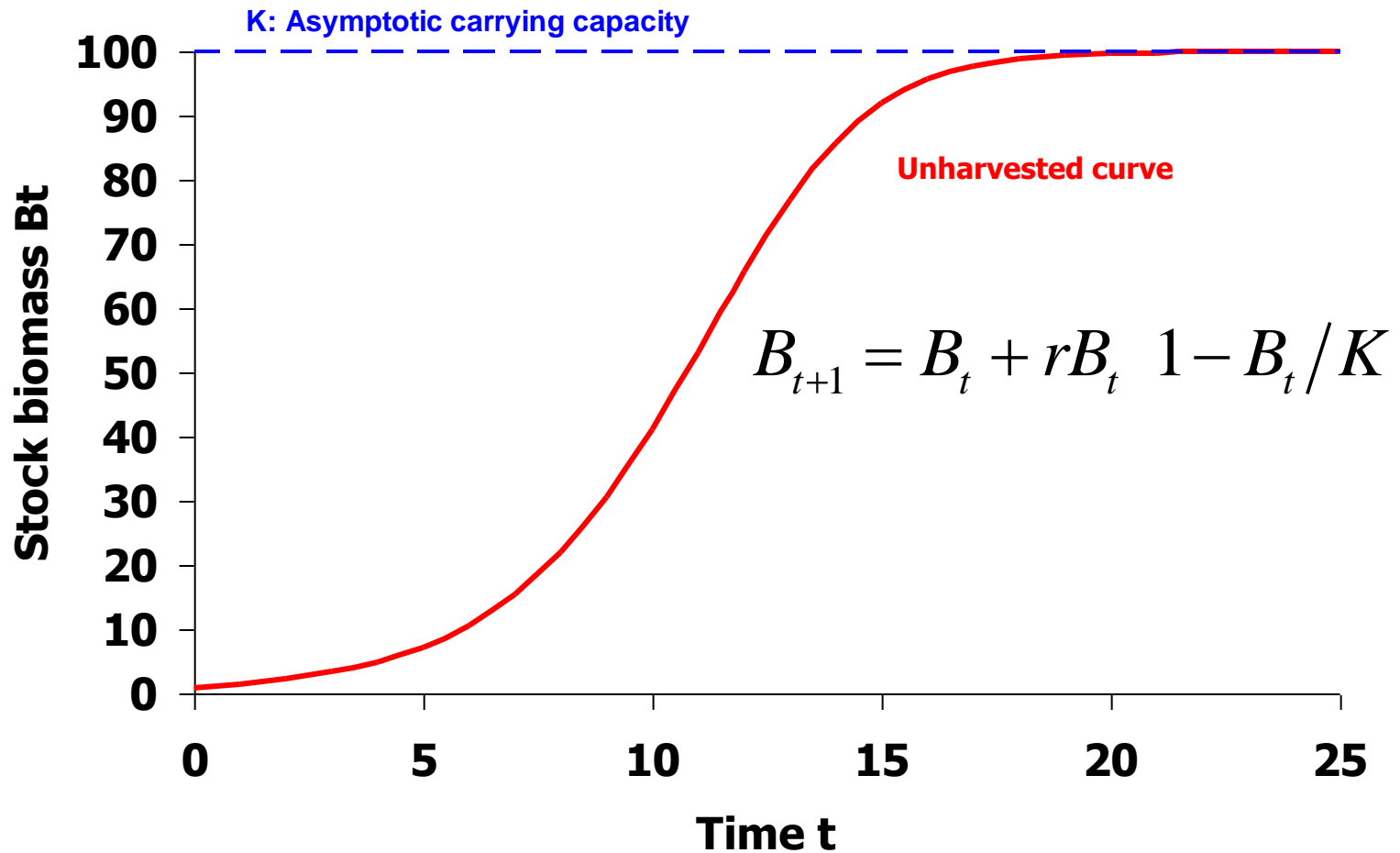
$$f B_t = rB_t \left(1 - \frac{B_t}{K} \right)$$

- The more general Pella & Tomlinson form:

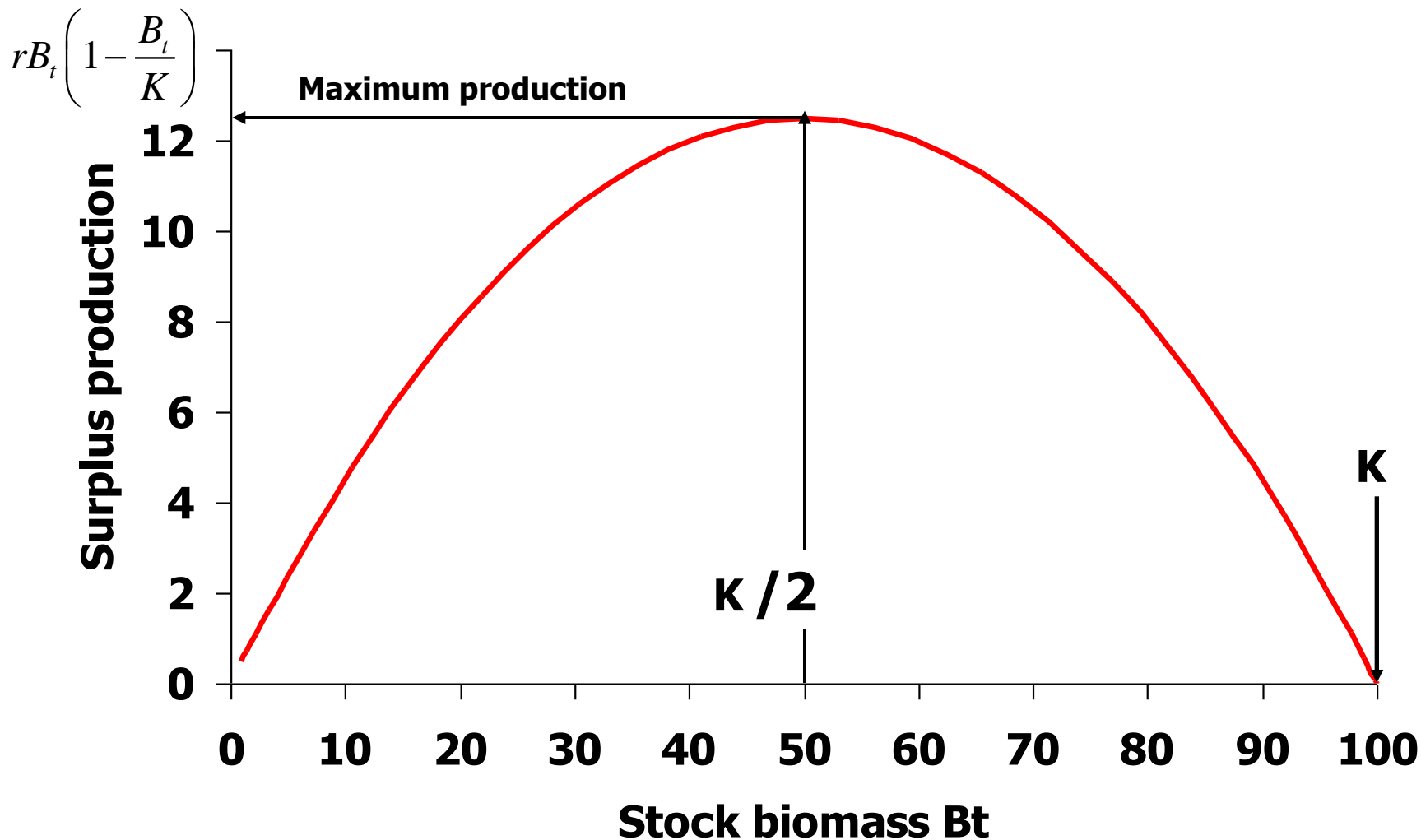
$$f B_t = \frac{r}{p} B_t \left(1 - \left[\frac{B_t}{K} \right]^p \right)$$

- Note: when $p=1$ the two functional forms are the same

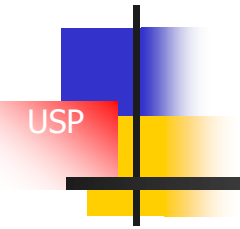
Population trajectory according to Schaefer model



Surplus production as a function of stock size



- Once the parameters have been estimated, fishery performance indicators useful to fisheries management can be calculated.
 - Biomass giving maximum sustainable yield: $B_{MSY} = K/2$
 - Maximum sustainable yield: $MSY = rK/4$
 - Effort that should lead to MSY: $E_{MSY} = r/2q$
 - Fishing mortality rate at MSY: $F_{MSY} = r/2$



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Methods of fitting

Estimation procedures

- All methods rely on the assumption that: $CPUE_t = f B_t$
 - Normally assume:
$$CPUE_t = \frac{C_t}{E_t} = qB_t$$
 - Equilibrium methods
 - Never use equilibrium methods!
 - Regression (linear transformation) methods
 - Computationally quick
 - Often make odd assumption about error structure
 - Time series fitting
 - Currently considered to be best method available

Often use:

$$U_t = CPUE_t$$

The equilibrium model 1

- The model:

$$B_{t+1} = B_t + rB_t \left(1 - \frac{B_t}{K}\right) - C_t \quad CPUE_t = \frac{C_t}{E_t} = qB_t$$

- The equilibrium assumption

$$\text{Catch}_t = \text{Production}$$

$$B_{t+1} = B_t \text{ and thus } C_t = rB_t \left(1 - \frac{B_t}{K}\right)$$

- Each years catch and effort data represent a an equilibrium situation where the catch is equal to the surplus production at that level of effort
- If the fishing regime is changed (effort or harvest rate) the stock is assumed to move instantaneously to a different stable equilibrium biomass with associated surplus production. The time series nature of the data is thus ignored.
- THIS IS PATENTLY WRONG, SO NEVER FIT THE DATA USING THE EQUILIBRIUM ASSUMPTION

The equilibrium model 2

- Given this assumption: $C_t = rB_t \left(1 - \frac{B_t}{K}\right)$
- It can be shown that:

$$B_{t+1} = B_t + rB_t \left(1 - \frac{B_t}{K}\right) - Y_t \quad CPUE_t = \frac{C_t}{E_t} = qB_t$$

- Can be simplified to

$$\frac{C}{E} = a - bE$$

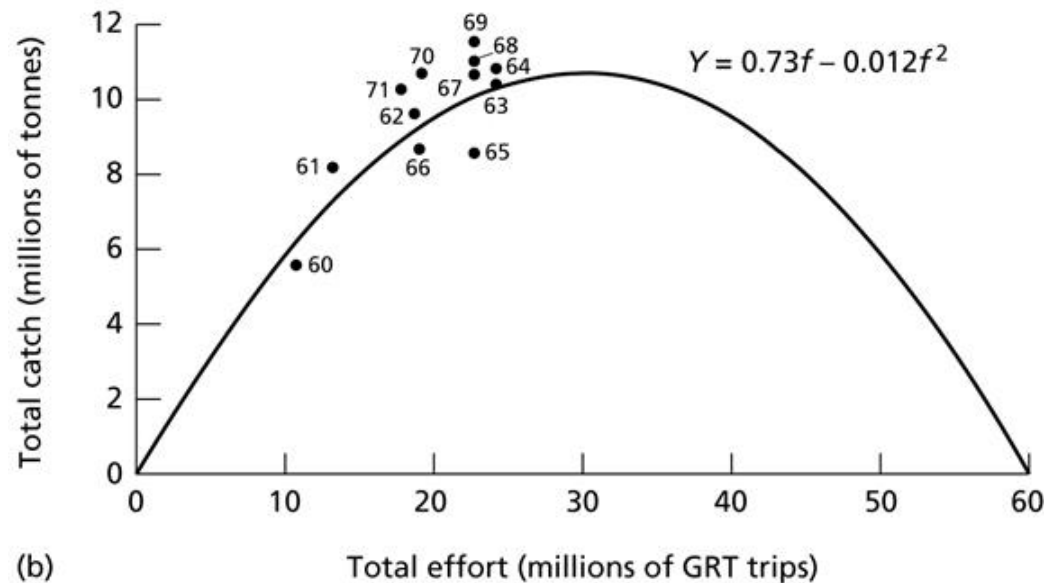
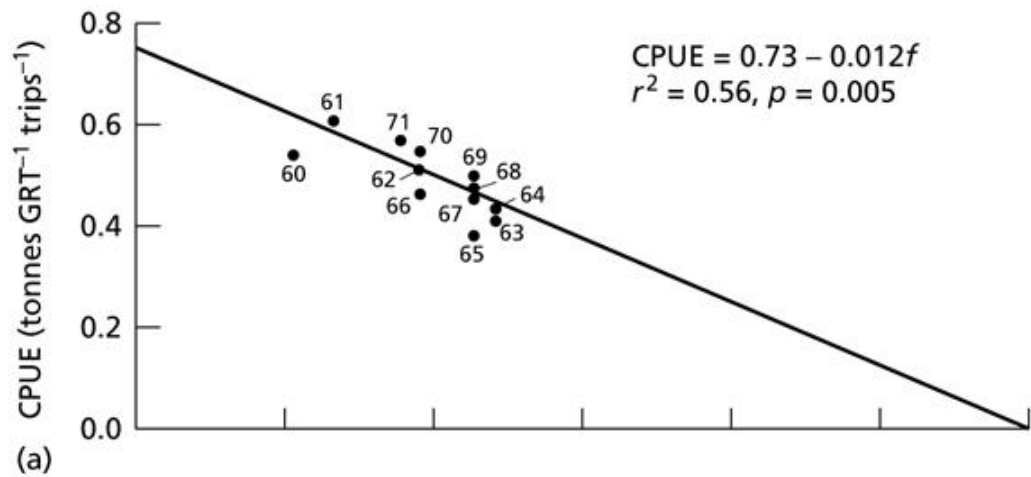
$$C = aE - bE^2$$

- Where

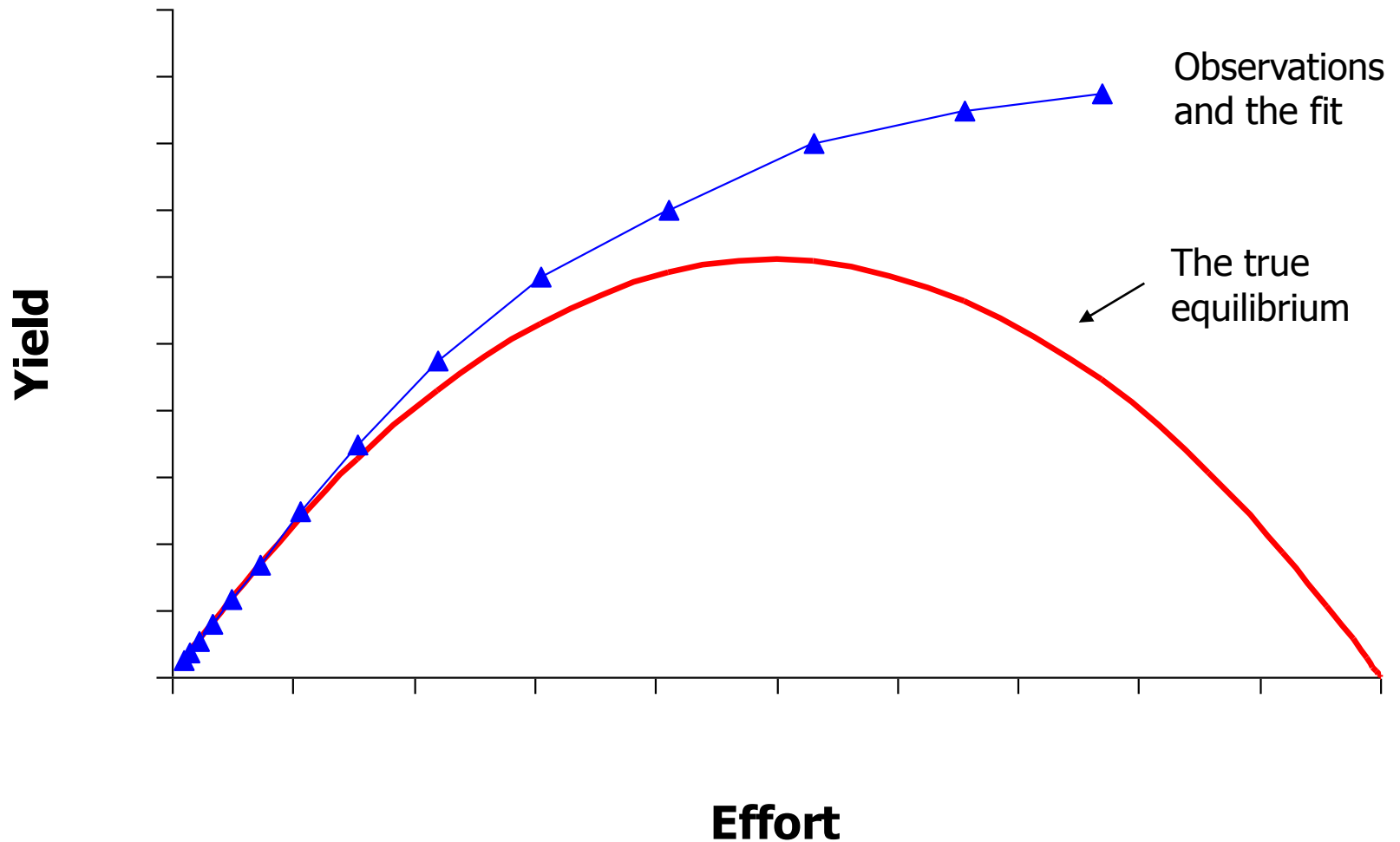
$$E_{MSY} = a/(2b)$$

$$MSY = (a/2)^2/b$$

The equilibrium model 3



If effort increases as the fisheries develops we may expect to get data like the blue trajectory



The regression model 1

Next biomass = this biomass + surplus production - catch

$$\boxed{1} \quad B_{t+1} = B_t + rB_t \left(1 - \frac{B_t}{K}\right) - qE_t B_t \quad \leftarrow Y_t = F_t B_t = qE_t B_t$$

$$\boxed{2} \quad B_t \stackrel{CPUE_t}{=} \frac{U_t}{q}$$

• Substituting $\boxed{2}$ in $\boxed{1}$ gives:

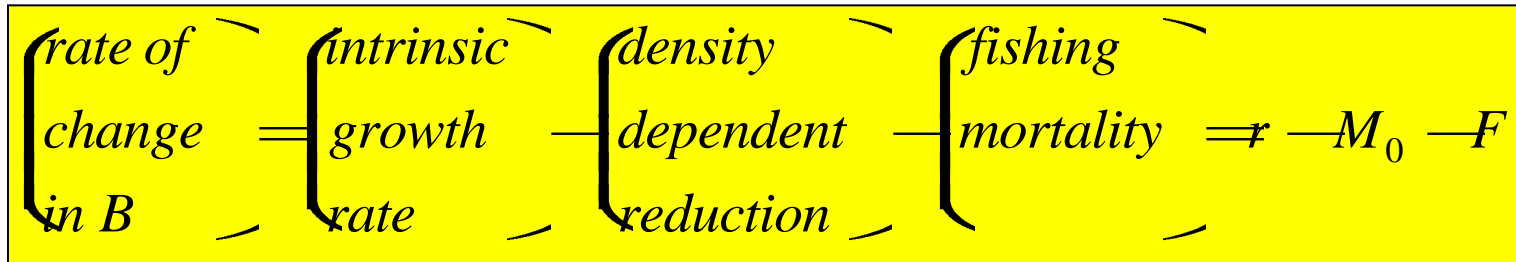
$$\frac{U_{t+1}}{q} = \frac{U_t}{q} + r \frac{U_t}{q} \left(1 - \frac{U_t}{qB_\infty}\right) - E_t U_t \quad \text{.....and dividing by } \frac{U_t}{q}$$

$$\Rightarrow \frac{U_{t+1}}{U_t} = 1 + r - \frac{rU_t}{qK} - E_t q \quad \Rightarrow \frac{U_{t+1}}{U_t} - 1 = r - \frac{r}{qK} U_t - qE_t$$

The regression model 2

$$\frac{U_{t+1}}{U_t} - 1 = r - \frac{r}{qK} U_t - qE_t$$

...this means:



Regress: $\frac{U_{t+1}}{U_t} - 1$ on U_t and E_t which is a multiple regression of the form:

$$Y = b_0 + b_1 X_1 + b_2 X_2 \quad \text{where} \quad X_1 = U_t \quad \text{and} \quad X_2 = E_t$$

$$b_0 = r, \quad b_1 = \frac{-r}{qK}, \quad b_2 = -q \Rightarrow K = \frac{-b_0}{b_1 \cdot b_2}$$

- The population model:

$$B_{t+1} = B_t + rB_t \left(1 - \frac{B_t}{K} \right) - Y_t$$

- The observation model:

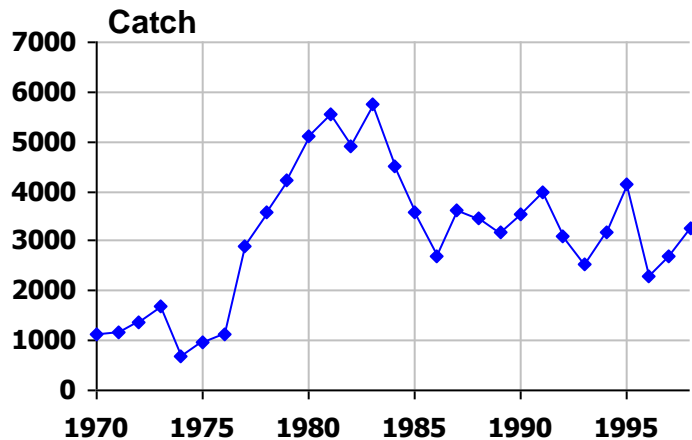
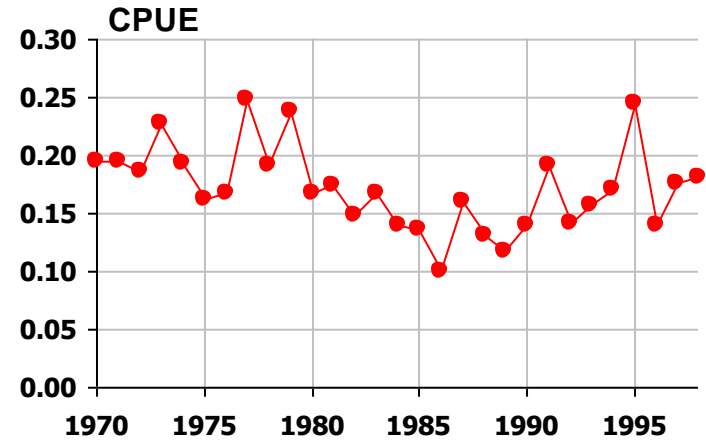
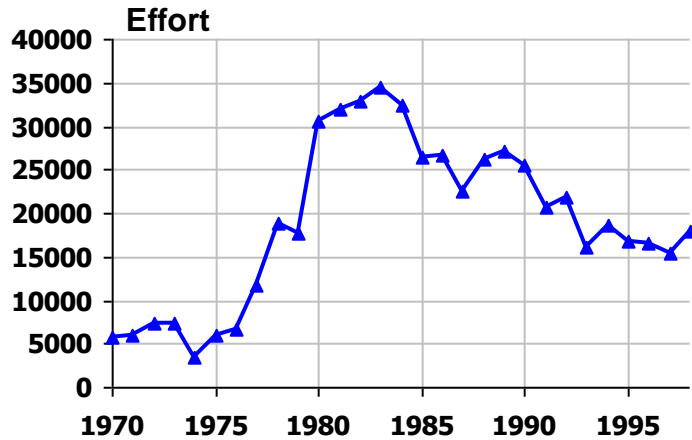
$$\hat{U}_t = qB_t + \varepsilon_i \text{ or } \hat{U}_t = qB_t e^{\varepsilon_i}$$

- The statistical model

$$\min SS = \sum_{t=0}^t U_t - \hat{U}_t^2 \text{ or } \min SS = \sum_{t=0}^t \ln U_t - \ln \hat{U}_t^2$$

- The population model:
 - Given some initial guesses of the parameters r and K and an initial starting value B_0 , and given the observed yield (Y_t) a series of expected B_t 's, can be produced.
- The observation model:
 - The expected value of B_t 's is used to produce a predicted series of \hat{U}_t (CPUE $_t$) by multiplying B_t with a catchability coefficient (q)
- The statistical model:
 - The predicted series of \hat{U}_t are compared with those observed (U_t) and the best set of parameter r, K, B_0 and q obtained by minimizing the sums of squares

Example: N-Australian tiger prawn fishery 1



History of fishery:

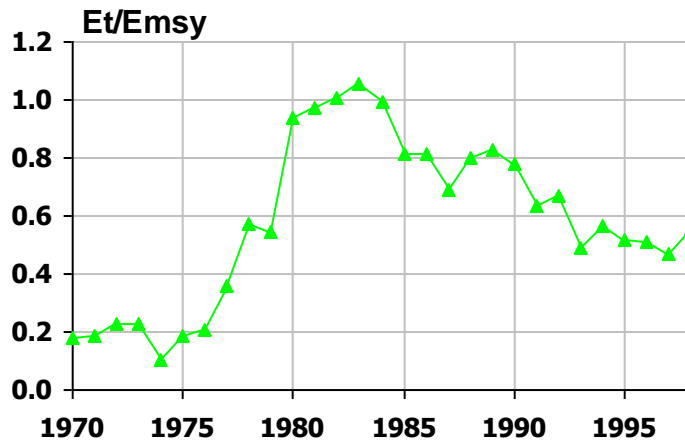
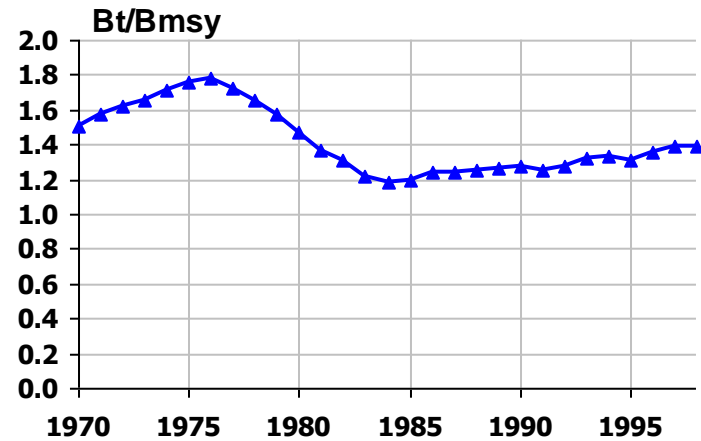
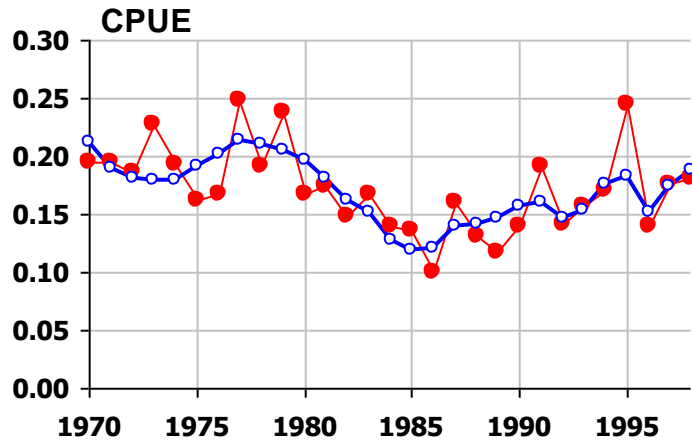
1970-76: Effort and catch low, cpue high

1976-83: Effort increased 5-7fold and catch 5fold, cpue declined by 3/4

1985-: Effort declining, catch intermediate, cpue gradually increasing.

Objective: Fit a stock production model to the data to determine state of stock and fisheries in relation to reference values.

Example: N-Australian tiger prawn fishery 2



The observe catch rates (red) and model fit (blue).

Parameters: $r = 0.32$, $K = 49275$, $B_0 = 37050$

Reference points:

MSY: 3900

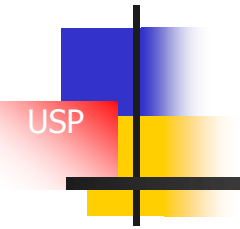
Emsy: 32700

Bcurrent/Bmsy: 1.4

The analysis indicates that the current status of the stock is above Bmsy and that effort is below Emsy.

Question is how informative are the data and how sensitive is the fit.

- In addition to yield and biomass indices data may have information on:
 - Stock biomass may have been un-fished at the beginning of the time series. I.e. $B_0 = K$. Thus possible to assume that
 - $U_0 = qB_0 = qK$
 - However data most often not available at the beginning of the fisheries
 - Absolute biomass estimates from direct counts, acoustic or trawl surveys for one or more years. Those years may be used to obtain estimates of q .
 - Prior estimates of r , K or q
 - q : tagging studies
 - r : basic biology or other similar populations
 - In the latter case make sure that the estimates are sound
 - K : area or habitat available basis

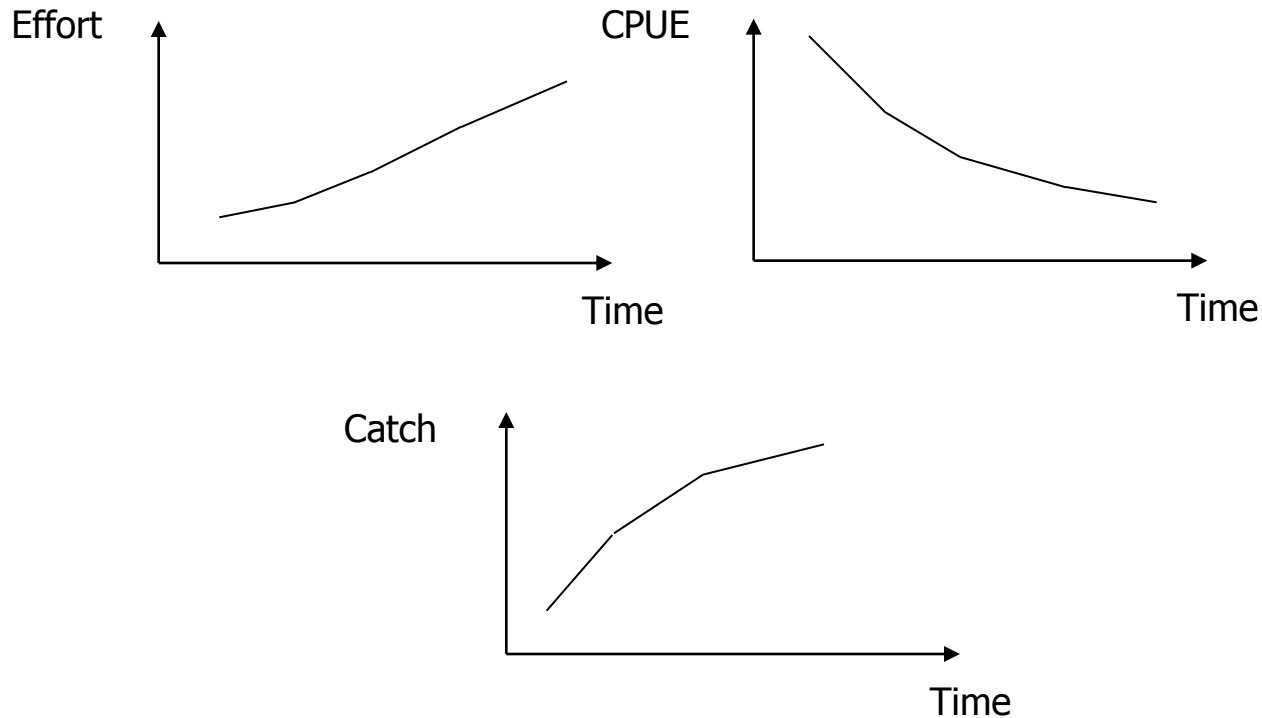


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Some word of caution

One way trip

- “One way trip”
 - Increase in effort and decline in CPUE with time
 - A lot of catch and effort series fall under this category.



Fit #	r	K	q	MSY	Emsy	SS
R	1.24	$3 \cdot 10^5$	$10 \cdot 10^{-5}$	100,000	$0.6 \cdot 10^6$	----
T1	0.18	$2 \cdot 10^6$	$10 \cdot 10^{-7}$	100,000	$1.9 \cdot 10^6$	3.82
T2	0.15	$4 \cdot 10^6$	$5 \cdot 10^{-7}$	150,000	$4.5 \cdot 10^6$	3.83
T3	0.13	$8 \cdot 10^6$	$2 \cdot 10^{-7}$	250,000	$9.1 \cdot 10^6$	3.83

- Identical fit to the CPUE series, however:
 - K doubles from fit 1 to 2 to 3
 - Thus no information about K from this data set
 - Thus estimates of MSY and Emsy is very unreliable

The importance of perturbation histories 1

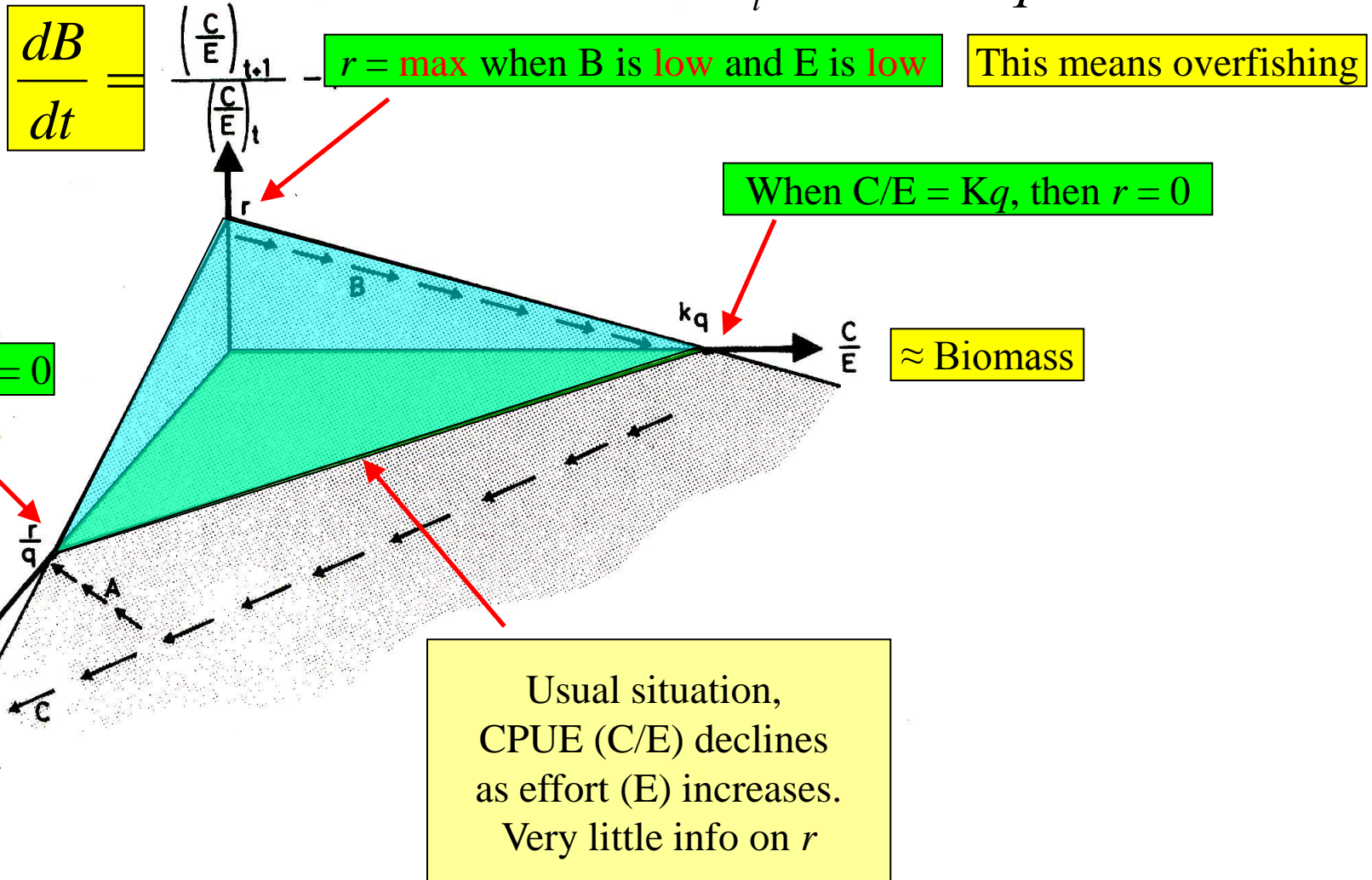
- “Principle: You can not understand how a stock will respond to exploitation unless the stock has been exploited”. (Walters and Hilborn 1992).
- Ideally, to get a good fit we need three types of situations:

<u>Stock size</u>	<u>Effort</u>	<u>get parameter</u>
low	low	r
high ($\rightarrow K$)	low	K (given we know q)
high/low	high	q (given we know r)

- Due to time series nature of stock and fishery development it is virtually impossible to get three such divergent & informative situations

The importance of perturbation histories 2

$$\frac{U_{t+1}}{U_t} - 1 = r - \frac{r}{qK} U_t - qE_t$$



One more word of caution

- Most abused stock-assessment technique!
 - Published applications based on the assumption of equilibrium should be ignored
 - Problem is that equilibrium based models almost always produce workable management advice. Non-equilibrium model fitting may however reveal that there is no information in the data.
 - Latter not useful for management, but it is scientific
- Efficiency is likely to increase with time, thus may have:

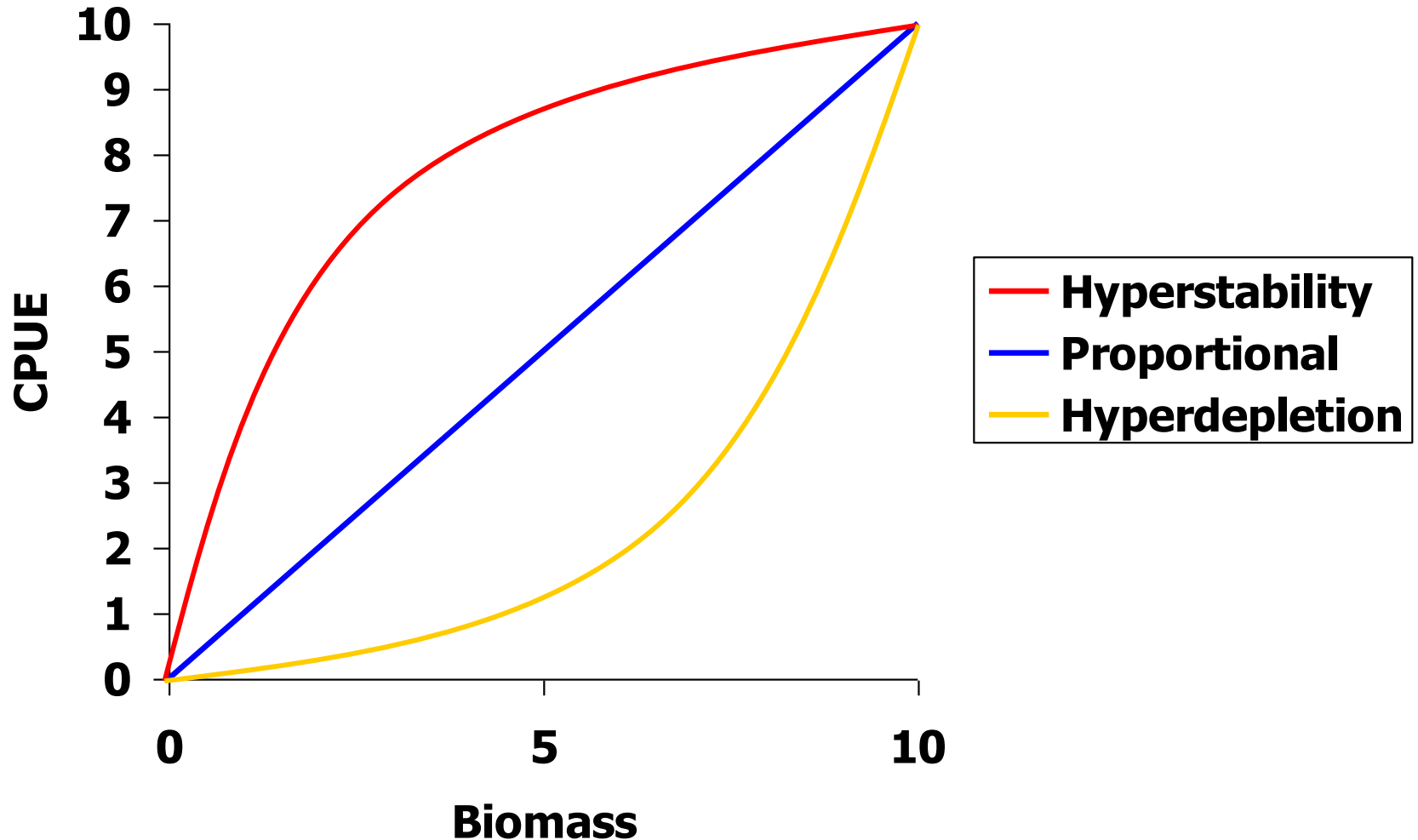
$$q_t = q_0 + q_{incr}t$$

- Commercial CPUE indices may not be proportional to abundance. I.e.

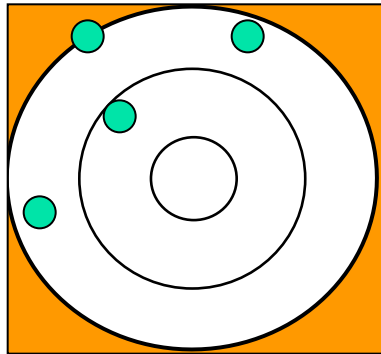
$$CPUE_t \neq qB_t$$

- Is the relationship is truly proportional?

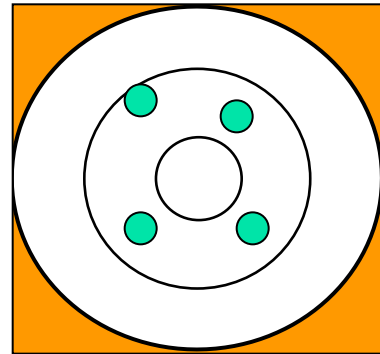
CPUE = f(B): What is the true relationship??



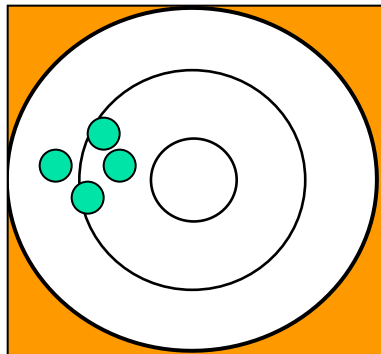
Accuracy = Precision + Bias



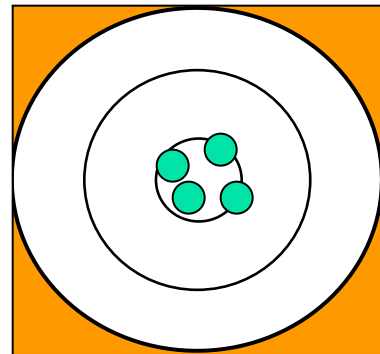
Not accurate and not precise



Accurate but not precise
(Vaguely right)



Precise but not accurate
(Precisely wrong)



Accurate and precise

It is better to be vaguely right than precisely wrong!